

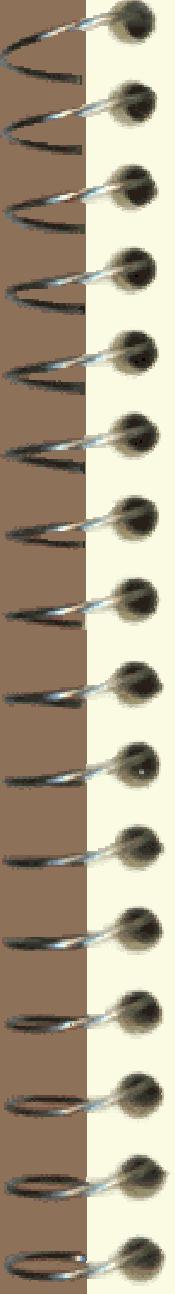
# Tangent Linear and Adjoint Models of Physics for a Global Spectral Model at NCEP/NOAA

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# My M.S. project:

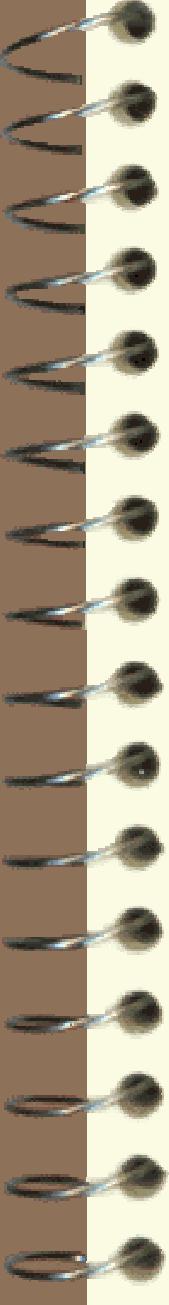
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Developing and testing of 2 software tools for  
NCEP's global spectral model physics

- ✓ Tangent Linear Model (TLM)
- ✓ Adjoint of TLM

Seminar will show:

- ✓ What TLM and adjoint are
- ✓ How they are used
- ✓ What I did



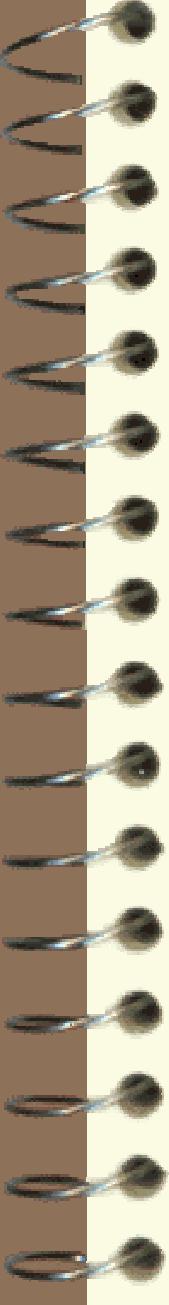
# Summary: (Remember this!)

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- ✓ Writing and testing TLM and adjoint are big jobs.
- ✓ A single integration of the adjoint model gives forecast sensitivity to initial condition or any parameter.

Reference:

Ronald M. Errico: What is an adjoint model? BAMS, Nov., 1997



# OUTLINE

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## Part I (Background): General Principles

- ✓ Defn of Tangent Linear (TLM) and adjoint model
- ✓ Applications of adjoint: sensitivity, data assim., etc.
- ✓ TLM and adjoint for examples.

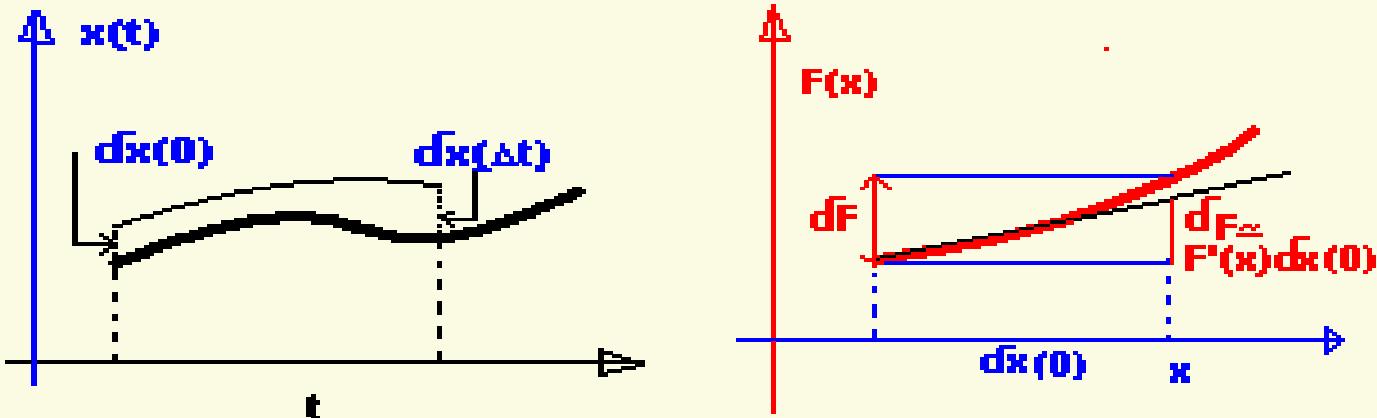
## Part II (My work): TLM & Adjoint of NCEP physics

- ✓ Complications of adjoints of NCEP physics
- ✓ Summary of NCEP parameterizations
- ✓ Developing and testing

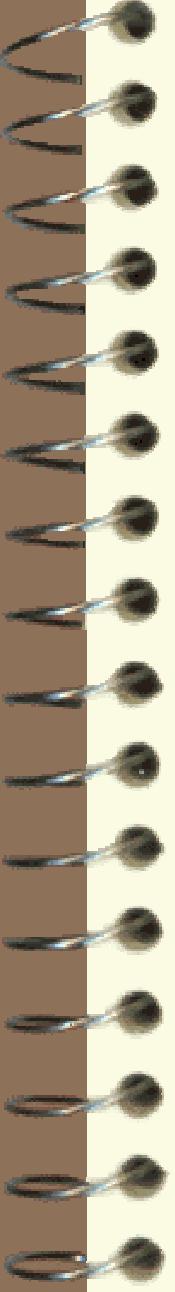
## Future Work

- ✓ Combining physics with dynamics

# What is a Tangent Linear Model (TLM)?



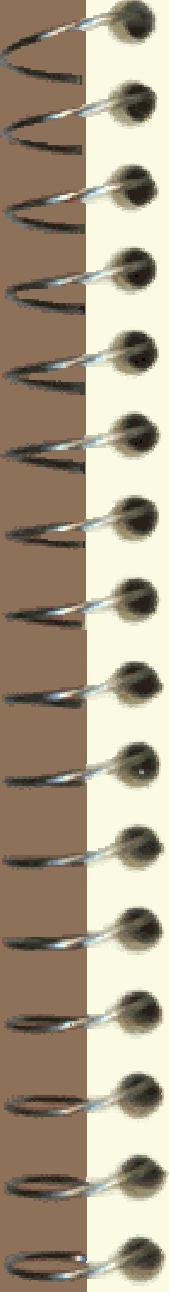
- ✓ Linearized version of nonlinear model around the basic state of model trajectory, describing perturbation evolution along the “tangent” direction of the nonlinear model.
- ✓ TL version of model  $\frac{\partial x}{\partial t} = F(x(t))$  is  $\frac{\partial \delta x}{\partial t} = F'(x)\delta x$   
$$\delta x(\Delta t) = \delta x(0) + F'(x)\delta x(0) \Delta t$$



# What is an inner product?

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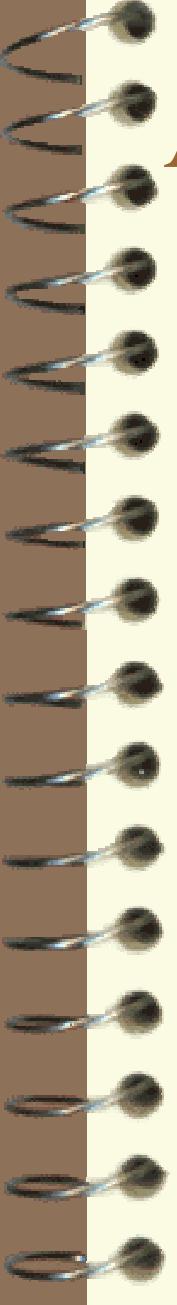
- ✓ Projects one “vector” onto another.
- ✓ Denoted  $\langle u, v \rangle$
- ✓ Dot product of vectors:  $\langle u, v \rangle = u^T v$
- ✓ Integral of functions:  $\langle u, v \rangle = \iint u v \, dx \, dy$



# What is an Adjoint?

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- ✓ Adjoint of linear operator  $L$  is  $L^*$  such that  $\langle Lu, v \rangle = \langle u, L^*v \rangle$  for any  $u, v$ . ( $L^*$  is not the complex conjugate of  $L$ .)
- ✓ If  $u$  and  $v$  are column vectors and  $L$  is a matrix, adjoint  $= L^* =$  transpose of  $L$ . If  $L = L_N \dots L_2 L_1$ , then  $L^* = L^T = L_1^T L_2^T \dots L_N^T$ .
- ✓ If  $u$  and  $v$  are functions and  $L$  is a differential operator, integrate  $\langle Lu, v \rangle$  by parts to get  $\langle u, L^*v \rangle$  and hence  $L^*$ .



# Adjoint Applications

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- ✓ Sensitivity analysis

Let  $R$  = any response function of the forecast

$f_j$  =  $j$ -th variable in the forecast

$u_i$  =  $i$ -th control variable (initial condition or  
model parameter)

$$\frac{\partial R}{\partial u_i} = \sum_j \frac{\partial R}{\partial f_j} \frac{\partial f_j}{\partial u_i}$$

$$= \sum_j \frac{\partial f_j}{\partial u_i} \frac{\partial R}{\partial f_j}$$

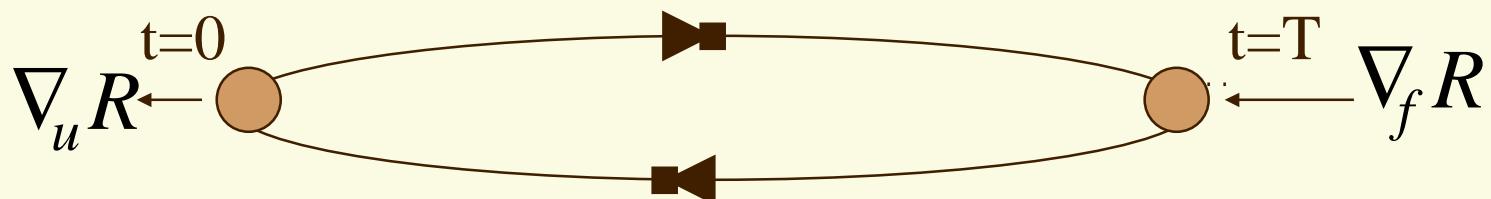
# Adjoint Applications

In mixed vector-matrix notation,

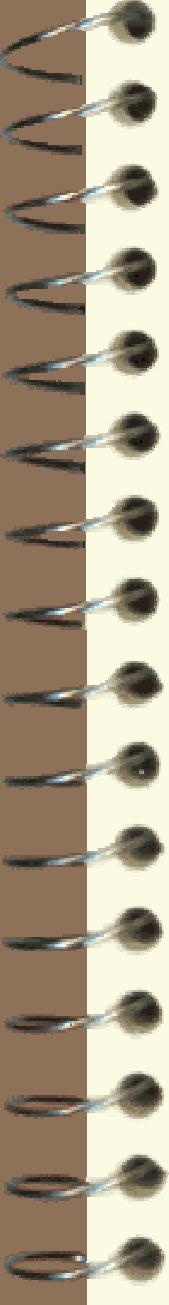
$$\nabla_u R = L^T \nabla_f R$$

$$= L^* \nabla_f R \quad \text{where} \quad L = \{L_{ij}\} = \left\{ \frac{\partial f_j}{\partial u_i} \right\}$$

Forward in time with Nonlinear Model to get  $f_j(t)$



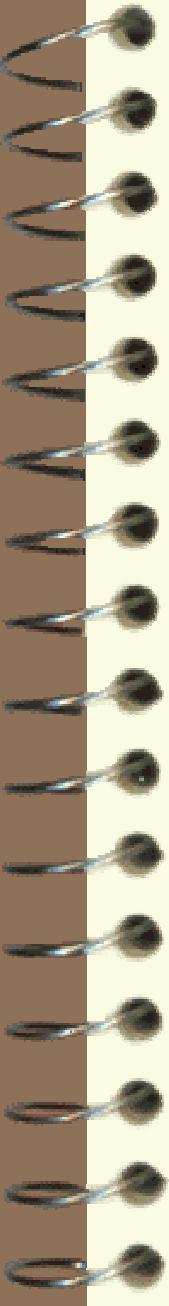
Backward in time with Adjoint of TLM to get  $\nabla_u R$



# Other Adjoint Applications

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- ✓ Data assimilation:
  - Navon, Zou, et al., 1992 (MWR)
  - Zupanski et al., 1995 (MWR)
- ✓ Parameter estimation.
  - Zou et al., 1992 (QJRMS)
  - Stauffer and Bao, 1993 (Tellus)
- ✓ Synoptic studies.
  - Vukicevic and Raeder, 1995 (MWR)
- ✓ ...



# Developing Tangent Linear Model (TLM) & Adjoint

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- ✓ Regular rules for differentiation -- simple examples
- ✓ One-dimensional advection equation

# Simple examples

Function	Tangent linear	Adjoint
$y = kx^2$	$\delta y = 2kx \delta x$	$\delta x^* = 2kx \delta y^*$
$y_1 = ax$ $y_2 = bx$	$\delta y_1 = a\delta x$ $\delta y_2 = b\delta x$	$\delta x^* = b\delta y_2^*$ $\delta x^* = a\delta y_1^* + \delta x^*$

# Form of the TLM and Adjoint

✓ Example: 1-dimensional advection equation

Nonlinear:

$$\frac{\partial u}{\partial t} = u \frac{\partial u}{\partial x}$$

Tangent linear (TL):

$$\frac{\partial \delta u}{\partial t} = \delta u \frac{\partial u}{\partial x} + u \frac{\partial \delta u}{\partial x}$$

Adjoint of TL(\*):

$$-\frac{\partial \delta u^*}{\partial t} = \frac{\partial u}{\partial x} \delta u^* - \frac{\partial u \delta u^*}{\partial x}$$

# Discrete Versions for $\delta u_{x,t}$

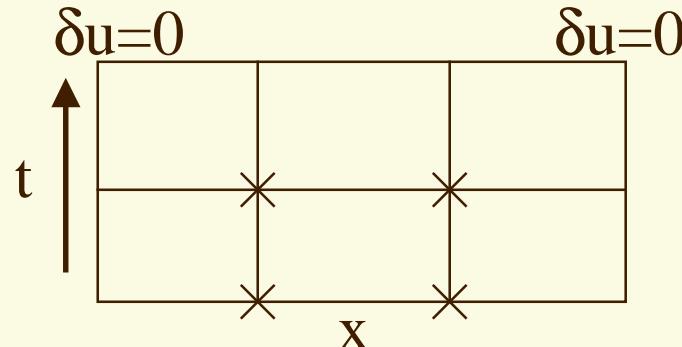
Tangent Linear (TL):

$$\left( \frac{\partial \delta u}{\partial t} = \delta u \frac{\partial u}{\partial x} + u \frac{\partial \delta u}{\partial x} \right)$$

Adjoint from TL:

$$\left( -\frac{\partial \delta u^*}{\partial t} = \frac{\partial u}{\partial x} \delta u^* - \frac{\partial u \delta u^*}{\partial x} \right)$$

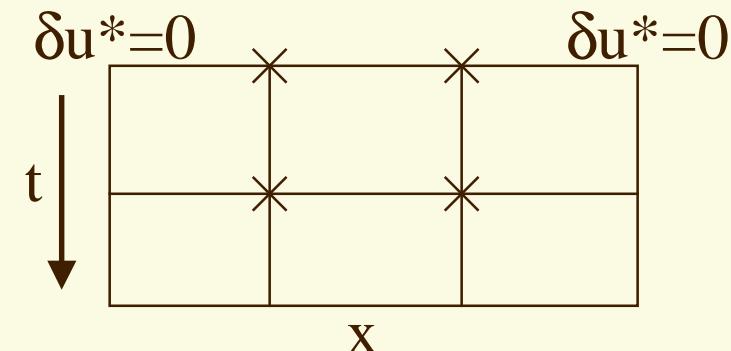
Adjoint from (\*):



$$\begin{pmatrix} \delta u_{23} \\ \delta u_{33} \end{pmatrix} = L_2 L_1 \begin{pmatrix} \delta u_{21} \\ \delta u_{31} \end{pmatrix}$$

$$\begin{pmatrix} \delta u_{21}^* \\ \delta u_{31}^* \end{pmatrix} = L_1^T L_2^T \begin{pmatrix} \delta u_{23}^* \\ \delta u_{33}^* \end{pmatrix}$$

$$\begin{pmatrix} \delta u_{21}^* \\ \delta u_{31}^* \end{pmatrix} = \Gamma_1 \Gamma_2 \begin{pmatrix} \delta u_{23}^* \\ \delta u_{33}^* \end{pmatrix}$$



# Test and Verification of TLM and Adjoint

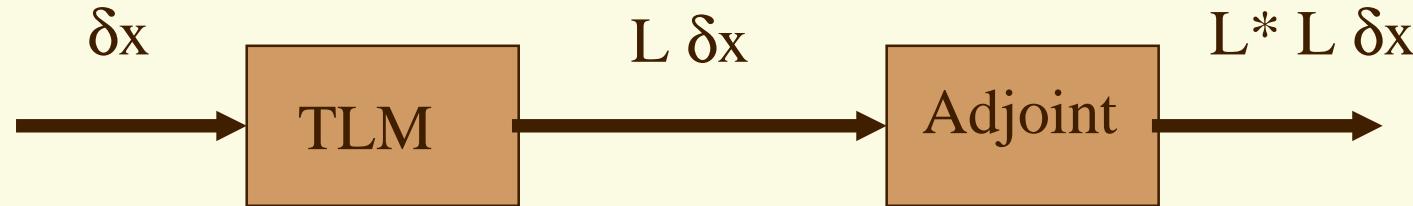
- ✓ Test of TLM

$$Ratio = \frac{\| F(u + \delta u) - F(u) \|}{\| F'(u) \delta x \|} = \frac{\| F(u + \delta u) - F(u) \|}{\| L \delta u \|}$$

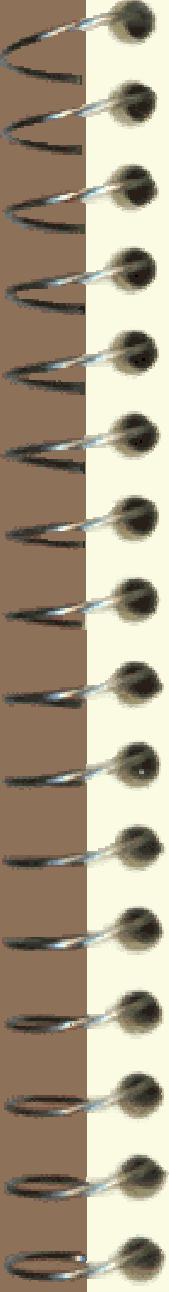
$\rightarrow 1 \text{ as } \| \delta x \| \rightarrow 0$

- ✓ Adjoint verification

$$\langle L \delta u, (L \delta u) \rangle = \langle \delta u, L^*(L \delta u) \rangle$$



- ✓ Gradient test

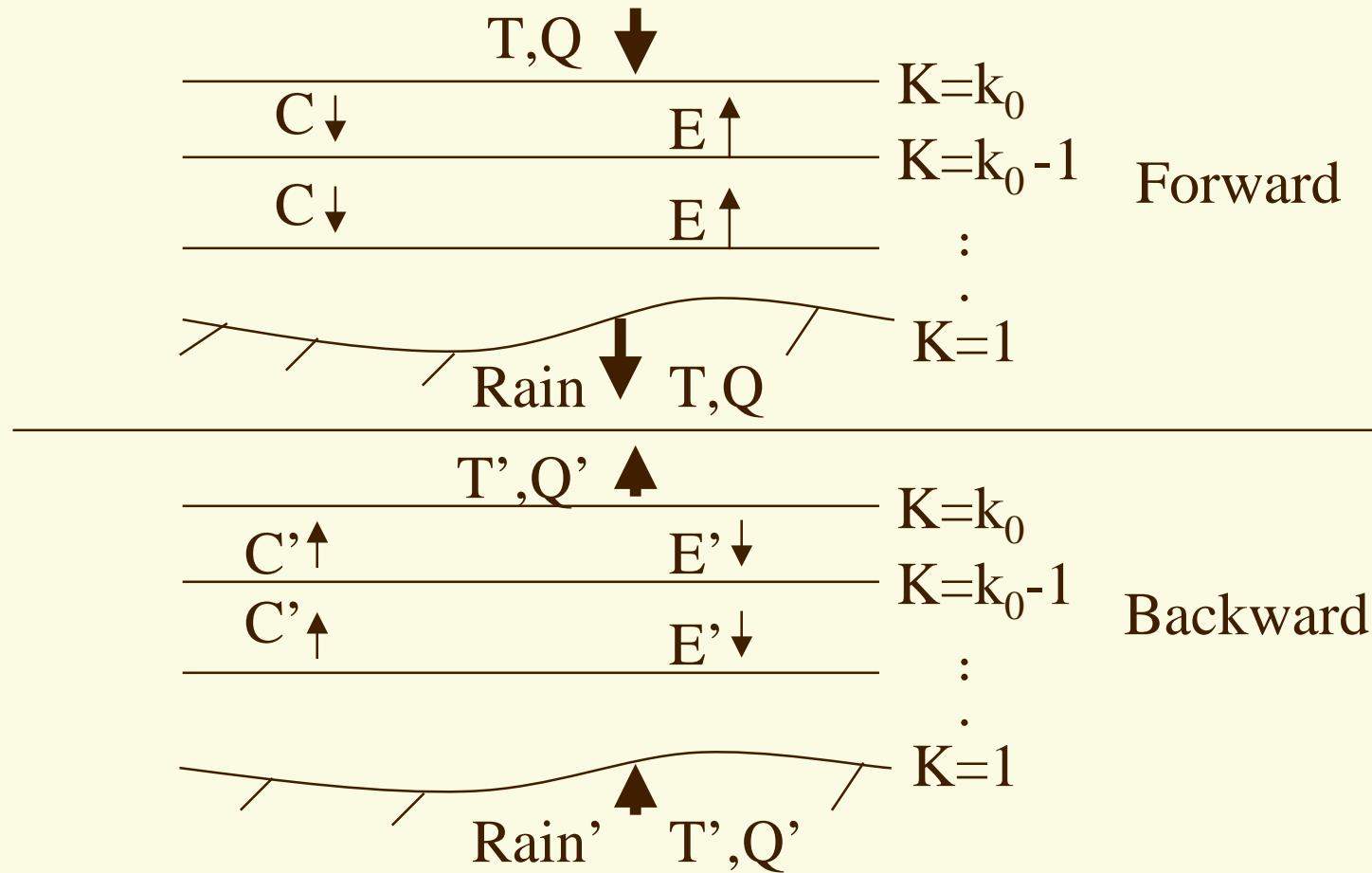


## Part II (My work): TLM & Adjoint of NCEP Physics

- ✓ Complications of adjoints of NCEP physics
- ✓ Summary of NCEP physics and their TLMs & adjoints
- ✓ Test results for TLMs when  
 $\delta u = \alpha(u - \bar{u})$  for  $\alpha = 10^{-n}$ ,  $n = 1$  to 12
- ✓ Verification results for adjoints when  
 $\delta u = \alpha(u - \bar{u})$  for  $\alpha = 0.01$

# Adjoint of Physics

✓ Grid-scale condensation as an example



# Summary of physics and their TLM & adjoint

Processes	Scheme	Routine Size (lines)		
		NL	TL	Adj.
Grid-scale Condensation	Condensation and Evapouration	41	83	94
Cumulus Convection	Simplified Arakawa-Schubert (1974)	1227	2075	1957
Shallow Convection	Betts(1986)	150	174	284
Vertical Diffusion	Nonlocal-K diffusion Chang(1992)	576	724	1164
Gravity-Wave Drag	Pierrehumbert Scheme (1986)	754	1058	865
Boundary and Surface	Monin and Obukhov Similarity theory Two level soil model	1266	1937	1862

# Test results for TLM ( $\alpha = 10^{-n}$ )

# Adjoint verification results $\alpha = 0.01$

Process	$\langle L_{du}, L_{du} \rangle / \langle d_{u}, L^* L_{du} \rangle$	Difference
Grid-scale Condensation	0.3039298248325E+07 0.3039298248325E+07	0.0000000000000 E+07
Cumulus Convection	0.1240862786251E+09 0.1240862786251E+09	0.0000000000000 E+09
Shallow Convection	0.1372874204986E+07 0.1372874204986E+07	0.0000000000000 E+07
Vertical Diffusion	0.3736668241314E+07 0.3736668241313E+07	0.0000000000001 E+07
Gravity-wave Drag	0.3068833559893E+09 0.3068833559893E+09	0.0000000000000 E+07
Boundary and Surface	0.4527065155976E+09 0.4527065155976E+09	0.0000000000000 E+09
Driver	0.4204278667792E+15 0.4204278667790E+15	0.0000000000002 E+15



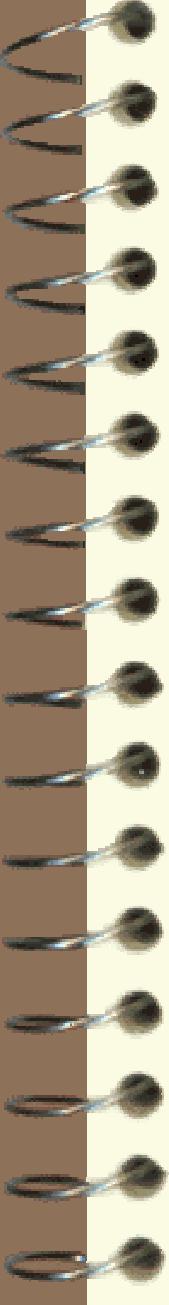
# Numerical Accuracy of Adjoint

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$\langle L\delta u, L\delta u \rangle = \langle \delta u, L^* L \delta u \rangle$  to 13 digits for:

- ✓ grid-scale condensation
- ✓ cumulus convection
- ✓ shallow convection
- ✓ gravity-wave drag
- ✓ boundary and surface processes

Differ by 1 or 2 in 13th digit for vertical diffusion  
and total package



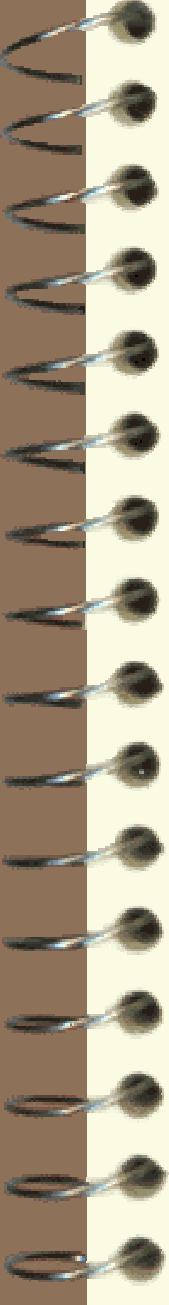
# Summary

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- ✓ TLM concept: linear evolution of a small perturbation while basic state evolves nonlinearly.
- ✓ From inner product to adjoint: $\langle Lu, v \rangle = \langle u, L^* v \rangle$
- ✓ Writing and testing TLM and adjoint are big jobs.
- ✓ Sensitivity-- fundamental application of adjoint

This seminar is in Web page form at:

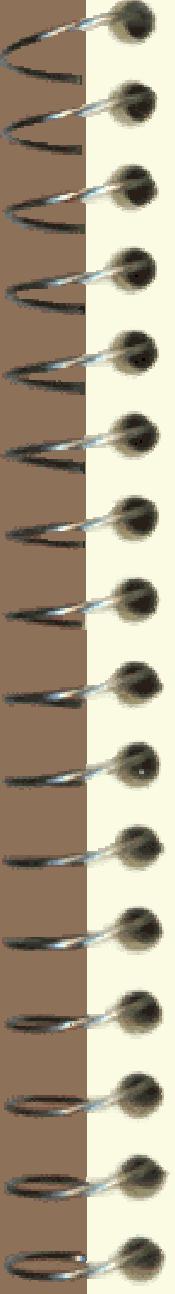
<http://www.gfdl.noaa.gov/~snz/mseminar.pdf>



# Future Work

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- ✓ Combine adjoints of physics and dynamics
- ✓ Test the whole model
- ✓ Jump point issue
- ✓ Data assimilation
- ✓ Sensitivity analysis
- ✓ Parameter estimation
- ✓ .....



# Thanks to .....

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My major professor ...

My committee: Drs. Navon, Pfeffer, & Zou

Drs. Sela & Kalnay and CITM ...

All friends ...

My wife ...

Questions?

